

# $Z^0$ Decay into Two Photons

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We calculate the decay rate of the  $Z^0 \rightarrow \gamma + \gamma$  process. The vertex of the  $\gamma^\mu \gamma_5$  is responsible for the decay of the weak vector boson into two photons, and the decay process becomes free from the Landau-Yang theorem. The calculated branching ratio is found to be  $\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma \simeq 2.4 \times 10^{-8}$ , in comparison with the present upper limit of the experimental branching ratio  $\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma < 5.2 \times 10^{-5}$ . This should be measurable by experiment as far as one can observe photons with around 45 GeV.

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## I. INTRODUCTION

The interaction between pions and nucleons can be described in terms of the pseudoscalar coupling as

$$\mathcal{L}_I = ig_\pi \bar{\psi} \gamma_5 \boldsymbol{\tau} \psi \cdot \boldsymbol{\varphi} \quad (1.1)$$

where  $g_\pi$  denotes the pion-nucleon coupling constant. Among pions,  $\pi^0$  decays into two photons where the decay rate can be described by the triangle Feynman diagrams which involve the fermion loops. This calculation is well explained in the textbook of Nishijima [1] and the decay width can be written as

$$\Gamma_{\pi^0 \rightarrow 2\gamma} = \frac{\alpha^2}{16\pi^2} \frac{g_\pi^2}{4\pi} \left( \frac{m_\pi}{M_N} \right)^2 m_\pi \quad (1.2)$$

where  $\alpha$ ,  $m_\pi$  and  $M_N$  denote the fine structure constant, the mass of pion and the mass of nucleon, respectively. If we take the value of the pion-nucleon coupling constant  $g_\pi$  as  $\frac{g_\pi^2}{4\pi} \simeq 8$  as suggested from the nucleon-nucleon scattering data [2], we can obtain the decay width of  $\Gamma_{\pi^0 \rightarrow 2\gamma} \simeq 7.5$  eV which should be compared with the observed value of 7.8 eV. In this calculation, we take the intermediate fermions as nucleon and therefore there is no problem of the evaluation of the Feynman diagrams. However, if we take the intermediate fermion states as quarks, then there are some difficulties in connection with the scattering states in the intermediate fermion propagator when evaluating the Feynman diagrams, and we will discuss it later.

In the same way, we can calculate the decay rate of the  $Z^0 \rightarrow 2\gamma$  process. In this case, the interaction Lagrangian density for the  $Z^0$  boson  $Z^\mu$  and fermions  $\psi_\ell$  can be written as [3]

$$\mathcal{L}_{II} = g_z \bar{\psi}_\ell \gamma^\mu \gamma_5 \psi_\ell Z_\mu - 0.06 g_z \bar{\psi}_\ell \gamma^\mu \psi_\ell Z_\mu \quad (1.3)$$

which is obtained from the standard model weak Hamiltonian with  $\sin^2 \theta_W = 0.235$ . Here, the first term in eq.(1.3) is important since the decay of the  $Z^0$  boson into two photons is described by the parity violating part. In this case, the decay rate of the  $Z^0 \rightarrow 2\gamma$  can be described by the triangle diagrams which are basically the same as the  $\pi^0$  decay process. It should be noted that, if the interaction is the vector coupling between the  $Z^0$  and the fermion, then the decay cannot occur due to the conservation of the spin and parity, and this is known as the Landau-Yang theorem [4, 5]. Here, the interaction is the one that violates the space reflection as given in eq.(1.3). In this case, the calculation can be carried out in a straight forward fashion, and we obtain the branching ratio of the  $Z^0 \rightarrow 2\gamma$  to the total width of the  $Z^0$  decay as

$$(\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma)_{\text{Pred.}} \simeq 2.4 \times 10^{-8}. \quad (1.4)$$

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This should be compared to the present upper limit of the experimental branching ratio [6]

$$(\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma)_{\text{Exp.}} < 5.2 \times 10^{-5} \quad (1.5)$$

and one sees that it is indeed possible to measure it as far as one can observe the very high energy  $\gamma$ -ray of 45 GeV.

It may be important to note that the present calculation clearly proves that there is no anomaly equation in the triangle diagrams. One can easily calculate and prove that the linear divergence term vanishes to zero due to the Trace evaluation before the momentum integration [7]. In addition, the logarithmic divergence also vanishes to zero completely due to the Trace and momentum integrations. The finite terms are left which are indeed quite similar to the  $\pi^0 \rightarrow 2\gamma$  decay rate. The detailed discussion about the anomaly equation is given in [8].

## II. $\pi^0 \rightarrow \gamma + \gamma$ PROCESS

Before going to the discussion of the  $Z^0 \rightarrow 2\gamma$  decay, we first review the calculation of the  $\pi^0 \rightarrow 2\gamma$  process [1]. The interaction Lagrangian density between fermion and pion  $\mathcal{L}_I$  can be given as eq.(1.1). In this case, the corresponding T-matrix for the  $\pi^0 \rightarrow 2\gamma$  reaction process can be written as

$$T_{\pi^0 \rightarrow 2\gamma} = ie^2 g_\pi \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[ (\gamma \epsilon_1) \frac{1}{\not{p} - M_N + i\varepsilon} (\gamma \epsilon_2) \frac{1}{\not{p} - \not{k}_2 - M_N + i\varepsilon} \gamma_5 \frac{1}{\not{p} + \not{k}_1 - M_N + i\varepsilon} \right] + (1 \leftrightarrow 2). \quad (2.1)$$

where  $\epsilon_1^\mu(\lambda_1)$  and  $\epsilon_2^\mu(\lambda_2)$  denote the two polarization vectors of photons with the polarizations of  $\lambda_1, \lambda_2$ . In this T-matrix calculation, the apparent linear and logarithmic divergences can be completely canceled out due to the Trace evaluation. Therefore, the corresponding T-matrix is indeed finite, and the decay width  $\Gamma_{\pi^0 \rightarrow 2\gamma}$  becomes

$$\Gamma_{\pi^0 \rightarrow 2\gamma} = \frac{1}{(2\pi)^5} \frac{1}{16M_N^2 m_\pi} e^4 g_\pi^2 \sum_{\lambda_1, \lambda_2} |\varepsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_1^\nu k_1^\rho k_2^\sigma|^2 = \frac{\alpha^2}{16\pi^2} \frac{g_\pi^2}{4\pi} \left( \frac{m_\pi}{M_N} \right)^2 m_\pi. \quad (2.2)$$

By putting the observed values of the parameters appearing in eq.(2.2), we find

$$\Gamma_{\pi^0 \rightarrow 2\gamma} \simeq 7.5 \text{ eV}. \quad (2.3)$$

The evaluation of the Feynman diagrams is carried out by choosing the nucleon intermediate states. Therefore, there appears no pole in the denominators of the T-matrix of eq.(2.1) since the nucleon mass is larger than the pion mass. Here we should make a comment that, if we choose the quark models to evaluate the triangle diagrams, then the corresponding T-matrix should have a possible contribution of the pole term in the denominator since the quark can be on the mass shell. This should give rise to the logarithmic divergence in the parameter integration, but this can be replaced by some average values of the parameters.

## III. $Z^0 \rightarrow \gamma + \gamma$ PROCESS

Now we can calculate the triangle Feynman diagrams which correspond to the  $Z^0$  decay into two photons. The interaction Lagrangian density between  $Z^0$  and fermions can be given in eq.(1.3), and therefore, the corresponding T-matrix for the triangle diagrams can be written as

$$T_{Z^0 \rightarrow 2\gamma} = g_z \sum_i e_i^2 \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[ \frac{1}{\not{p} - m_i + i\varepsilon} (\gamma \epsilon_1) \frac{1}{\not{p} - \not{k}_2 - m_i + i\varepsilon} (\gamma \epsilon_2) \frac{1}{\not{p} + \not{k}_1 - m_i + i\varepsilon} (\gamma \epsilon_v) \gamma_5 \right] + (1 \leftrightarrow 2) \quad (3.1)$$

where  $m_i$  and  $e_i$  denote the mass and charge of the corresponding fermions in the intermediate states, and  $\epsilon_v$  denotes the polarization vector of the  $Z^0$  boson.

### A. Decay Width with Intermediate Top Quark States

Here, we take the intermediate top quark state since it gives the largest contribution to the decay width. The evaluation of the T-matrix can be carried out in a straight forward way just in the same manner as the  $\pi^0 \rightarrow 2\gamma$  process. In order to avoid any confusions, we discuss the term by term in the integration of eq.(3.1).

### 1. Linear Divergence Term

The leading term in the integration of eq.(3.1) at the large momentum of  $p$  should have the following shape

$$T_{Z^0 \rightarrow 2\gamma}^{(1)} \simeq g_z e^2 \int \frac{d^4 p}{(2\pi)^4} \left[ \frac{\text{Tr} \{ \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \gamma_\rho \gamma_5 \} \epsilon_1^\mu \epsilon_2^\nu \epsilon_v^\rho}{(p^2 - s_0 + i\epsilon)^3} + (1 \leftrightarrow 2) \right]. \quad (3.2)$$

In this case, we can easily prove the following equation

$$\text{Tr} \{ \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \gamma_\rho \gamma_5 \} = -\text{Tr} \{ \not{p} \gamma_\nu \not{p} \gamma_\mu \not{p} \gamma_\rho \gamma_5 \}$$

and therefore we obtain

$$T_{Z^0 \rightarrow 2\gamma}^{(1)} \simeq g_z e^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - s_0 + i\epsilon)^3} [\text{Tr} \{ \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \gamma_\rho \gamma_5 \} + \text{Tr} \{ \not{p} \gamma_\nu \not{p} \gamma_\mu \not{p} \gamma_\rho \gamma_5 \}] \epsilon_1^\mu \epsilon_2^\nu \epsilon_v^\rho = 0. \quad (3.3)$$

In this respect, there is no linear divergence in the triangle diagrams and thus there exists no anomaly [9–11].

### 2. Logarithmic Divergence Term

The  $p^2$  term of the numerator in eq.(3.1) contains the apparent logarithmic divergence. However, we find that the logarithmic divergence term vanishes to zero in an exact fashion. First, we can calculate the Trace of the  $\gamma$ - matrices and find the following shape for the logarithmic divergence term  $T_{Z^0 \rightarrow 2\gamma}^{(0)}$  as

$$T_{Z^0 \rightarrow 2\gamma}^{(0)} \simeq g_z e^2 \int_0^1 dx \int_0^x dy \int \frac{d^4 p}{(2\pi)^4} \frac{F(p, x, y)}{(p^2 - s_0 + i\epsilon)^3} \quad (3.4)$$

where  $F(p, x, y)$  is written

$$F(p, x, y) = \text{Tr} \{ \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \gamma_\rho \gamma_5 \} + \text{Tr} \{ \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \gamma_\rho \gamma_5 \} + \text{Tr} \{ \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \gamma_\rho \gamma_5 \} \quad (3.5)$$

where  $a, b, c$  are given as

$$a = -k_1(1-x) - k_2(1-y), \quad b = -k_1(1-x) + k_2 y, \quad c = -k_1 x + k_2 y$$

After some tedious but straight forward calculation, we find that

$$T_{Z^0 \rightarrow 2\gamma}^{(0)} = 0 \quad (3.6)$$

and therefore there is no need of the renormalization since the triangle diagrams are indeed all finite.

## B. Finite Terms

Here, one sees that the triangle diagrams with the axial vector coupling have neither linear nor logarithmic divergences. This is proved without any regularizations, and the total amplitude of  $Z^0 \rightarrow 2\gamma$  decay process is indeed finite. Here, we present the calculated decay width via top quarks since its contribution is the largest among all the other fermions. The finite term of the T-matrix can be written as

$$T_{Z^0 \rightarrow 2\gamma} = g_z e^2 \int_0^1 dx \int_0^x dy \int \frac{d^4 p}{(2\pi)^4} \frac{A(x, y)}{(p^2 - s_0 + i\epsilon)^3} \quad (3.7)$$

where  $A(x, y)$  is given as

$$A(x, y) = -4im_t^2(x+1-y)(k_1^\alpha - k_2^\alpha)\varepsilon_{\mu\nu\rho\alpha}\epsilon_1^\mu\epsilon_2^\nu\epsilon_v^\rho.$$

Here  $m_t$  denotes the mass of the top quark. Therefore, the T-matrix becomes

$$T_{Z^0 \rightarrow 2\gamma} = -\frac{g_z}{6\pi^2} \left( \frac{2e}{3} \right)^2 (k_1^\alpha - k_2^\alpha)\varepsilon_{\mu\nu\rho\alpha}\epsilon_1^\mu\epsilon_2^\nu\epsilon_v^\rho \quad (3.8)$$

In this case, we can calculate the decay width and obtain

$$\Gamma_{Z^0 \rightarrow 2\gamma}^{\text{Top}} = 3 \left( \frac{2^2 \alpha}{3^2} \right)^2 \frac{2\alpha_z}{27\pi^2} M_{z_0} \quad (3.9)$$

where  $\alpha_z = \frac{g_z^2}{4\pi} \simeq 2.7 \times 10^{-3}$ . In addition, the decay width with intermediate lepton and light quark states can be calculated in the same way as the above process. However, we see that the decay rates of light fermions are smaller by one order of magnitude than the top quark contribution because their masses are much smaller than the mass of the  $Z^0$  boson. Therefore, we have neglected them in the present calculation, but the inclusion of the contributions of the light fermions should make the decay width larger than the present number given in eq.(3.9).

### C. Branching Ratio

Now, we can evaluate the branching ratio of  $\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma$ , but before the calculation of the branching ratio, we should write the decay width of the  $Z^0 \rightarrow \ell^+ + \ell^-$  process

$$\Gamma_{Z^0 \rightarrow \ell^+ \ell^-} \simeq \frac{1}{3} \alpha_z M_{z_0} \simeq 0.083 \text{ GeV} \quad (3.10)$$

where we assume that  $M_{z_0} \gg m_\ell$ . Therefore, the calculated branching ratio of  $(\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma)_{\text{Pred.}}$  becomes

$$(\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma)_{\text{Pred.}} = (\Gamma_{Z^0 \rightarrow \ell^+ \ell^-}/\Gamma) \times (\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma_{Z^0 \rightarrow \ell^+ \ell^-}) \simeq 2.4 \times 10^{-8} \quad (3.11)$$

where the experimental value of  $\Gamma_{Z^0 \rightarrow \ell^+ \ell^-}/\Gamma = 0.034$  is used. This predicted branching ratio can be compared to the experimental upper limit of the branching ratio of this decay process [6]

$$(\Gamma_{Z^0 \rightarrow 2\gamma}/\Gamma)_{\text{Exp.}} < 5.2 \times 10^{-5} \quad (3.12)$$

which is still consistent with zero decay rate. The predicted value is, in fact, three orders of magnitude smaller than the present upper limit of the observation, but we believe that it should be measurable even though photon with around 45 GeV energies should be quite new to the present experimental detectors.

## IV. CONCLUSIONS

We have presented a new calculation of the weak boson of  $Z^0$  into two photons. The branching ratio is three orders of magnitudes smaller than the present experimental upper limit. This observation of the decay process must be very important in many respects. The measurement of the  $Z^0 \rightarrow 2\gamma$  should prove that the renormalization scheme works well for the weak decay process as well. In addition, it should prove experimentally that there is no anomaly equation present in any of the field theory models.

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